Class XI - MATHEMATICS

Chapter 1 – SETS

Module – 2/2

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Distance Learning Programme: An initiative by AEES, Mumbai

Learning Outcome

In this module we are going to learn about

- **Intervals as subset of R**
- **Power set**
- **Universal set**
- **Venn diagrams**
- **Union of sets**
- **Intersection of sets**
- **Practical problems on Union & Intersection of sets**

INTERVALS AS SUBSET OF REAL NUMBERS

Open Interval: Let $a, b \in R$ and $a < b$. Then the set of real numbers $\{ x : a < x < b\}$ is called an open interval and is denoted by (a, b).

Closed Interval: Let $a, b \in R$ and $a < b$. Then the set of real numbers $\{x : a \leq x \leq b\}$ is called closed interval and is denoted by $[a, b]$.

TYPES OF INTERVALS

- $(a, b) = \{ x : a < x < b, x \in R\}$
- $[a, b] = \{x : a \leq x \leq b, x \in R\}$
- $(a, b] = \{ x : a < x \leq b, x \in R\}$
- $[a, b) = \{ x : a \leq x < b, x \in R\}$
- $(0, \infty) = \{ x : 0 < x < \infty, x \in R\}$
- $(-\infty,\infty) = \{x: -\infty < x < \infty, x \in R\}$ = the set of real numbers R.

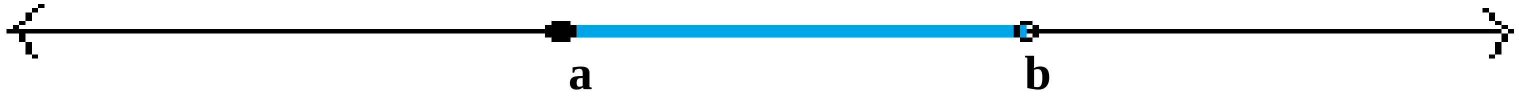
REPRESENTING INTERVALS ON NUMBER LINE

(a, b) ⇒

[a, b] ⇒

(a, b] ⇒

[a, b) ⇒



POWER SET

The collection of all subsets of a set A is called the power set of A. It is denoted by P(A).

if A = { 1, 2 }, then P(A) = { ϕ,{ 1 }, { 2 }, { 1,2 }}

In general, if A is a set with n(A) = m, then n [P(A)] = 2^m.

UNIVERSAL SET

In a particular context, we have to deal with the elements and subsets of a basic set which is relevant to that particular context. This basic set is called the "Universal Set".

The universal set is usually denoted by U, and all its subsets by the letters A, B, C, etc.

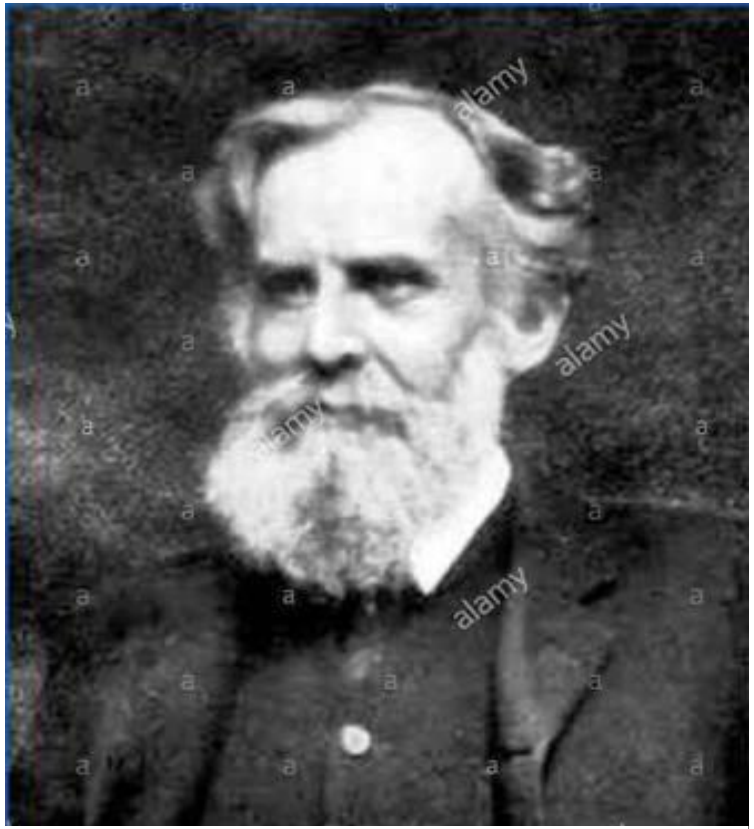
For example, for the set of all integers, the universal set can be the set of rational numbers or the set R of real numbers.

VENN DIAGRAMS

Most of the relationships between sets can be represented by means of diagrams which are known as Venn diagrams.

Venn diagrams are named after the English logician, John Venn (1834-1883).

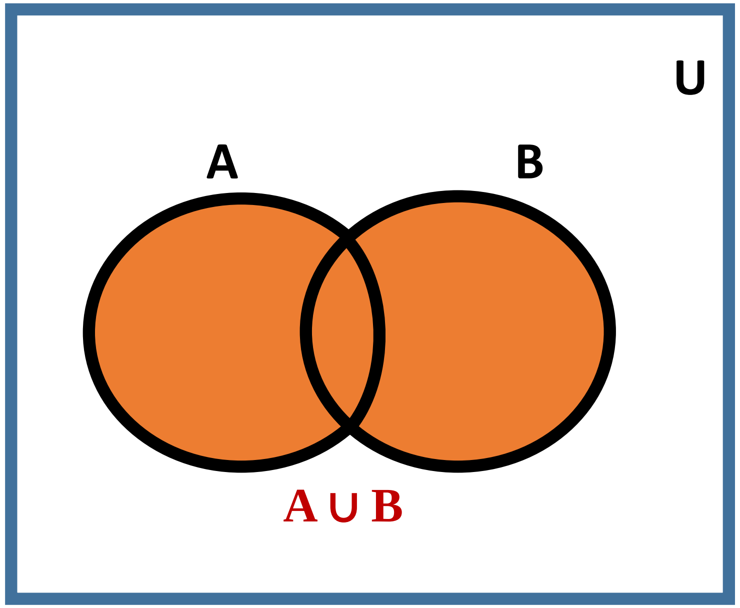
In a Venn diagram, the universal set is represented usually by a rectangle and its subsets by circles or ellipses.



John Venn (1834-1883).

UNION OF SETS

The union of two sets A and B is the set C which consists of all those elements which are either in A or in B (including those which are in both).



The union of two sets A and B is denoted as $A \cup B$ and usually read as 'A union B'. Hence, $A \cup B = \{ x : x \in A \text{ or } x \in B \}$

Some examples on union of two sets

Example 1 :

Let $A = \{ 2, 4, 6, 8\}$ and $B = \{ 6, 8, 10, 12\}$. Find $A \cup B$

Solution: $A \cup B = \{ 2, 4, 6, 8, 10, 12\}$

Example 2 :

Let $A = \{ a, b, c, d \}$ and $B = \{ a, c\}$. Find $A \cup B$

Solution: $A \cup B = \{a, b, c, d \}$

Note : If, $B \subset A$, then $A \cup B = A$.

PROPERTIES OF UNION OF TWO SETS

i) **$A \cup B = B \cup A$ (Commutative law)**

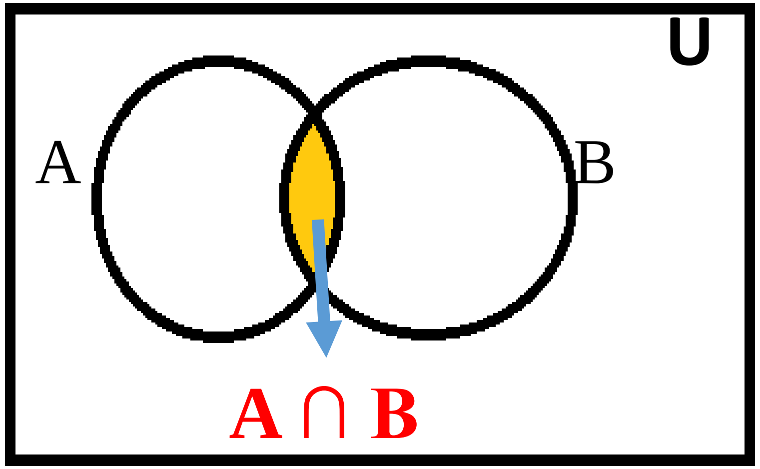
ii) $(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)

iii) $A \cup \emptyset = A$ (Law of identity element, $\emptyset$ is the identity of $\cup$)

iv) $A \cup A = A$ (Idempotent law)

v) $U \cup A = U$ (Law of U)

INTERSECTION OF TWO SETS



The intersection of two sets A and B is the set of all those elements which belong to both A and B.

Symbolically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$.

Example:

Let $A = \{1, 3, 4, 5, 6, 7, 8\}$ and $B = \{2, 3, 5, 7, 9\}$, find $A \cap B$.

Solution: $A \cap B = \{3, 5, 7\}$.

Note: If $A \subset B$, then $A \cap B = B$

PROPERTIES OF INTERSECTION OF TWO SETS

i) $A \cap B = B \cap A$ (Commutative law).

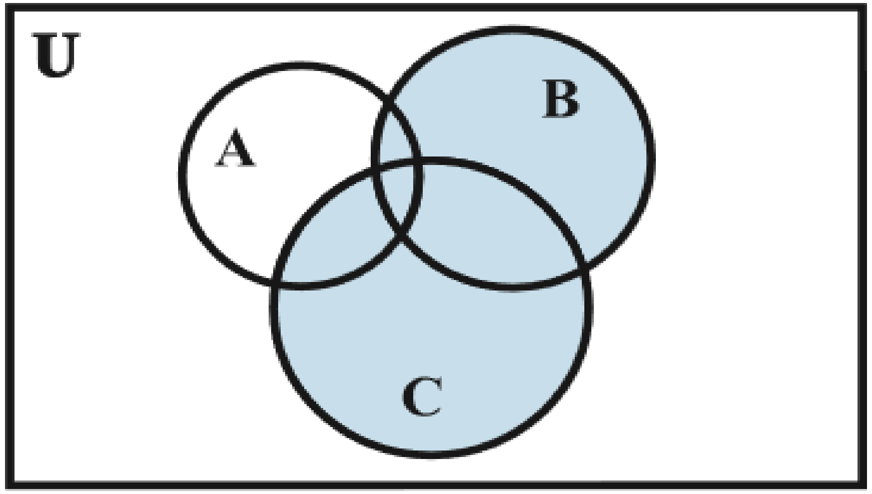
ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law).

iii) $\emptyset \cap A = \emptyset$, $U \cap A = A$ (Law of $\emptyset$ and U).

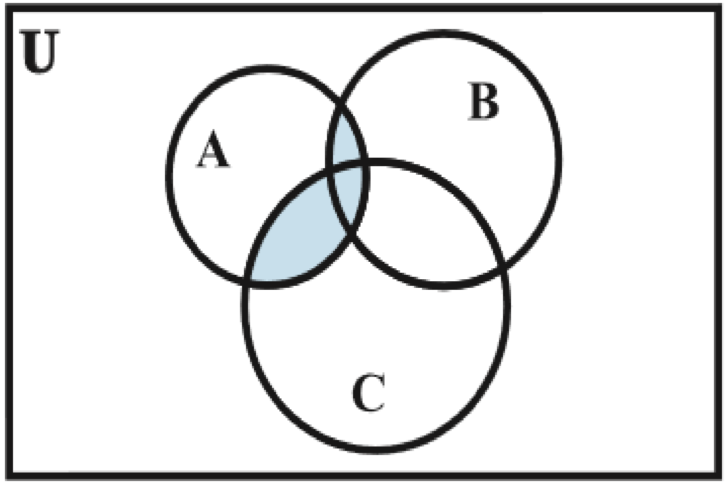
iv) $A \cap A = A$ (Idempotent law)

v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law)

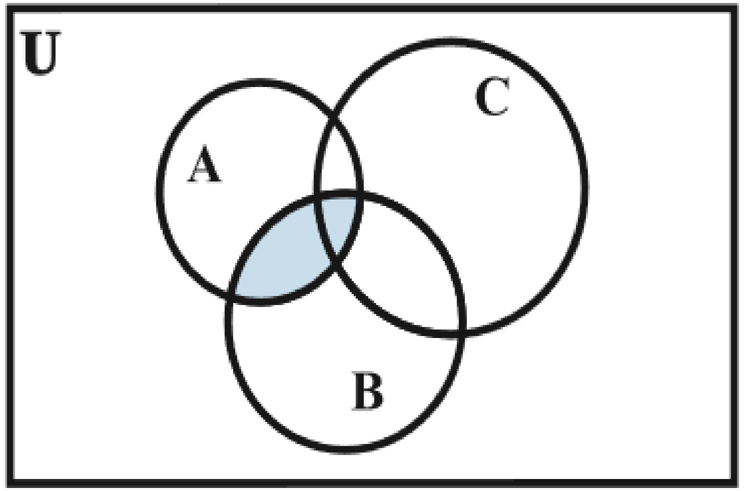
Distribution of Intersection over Union



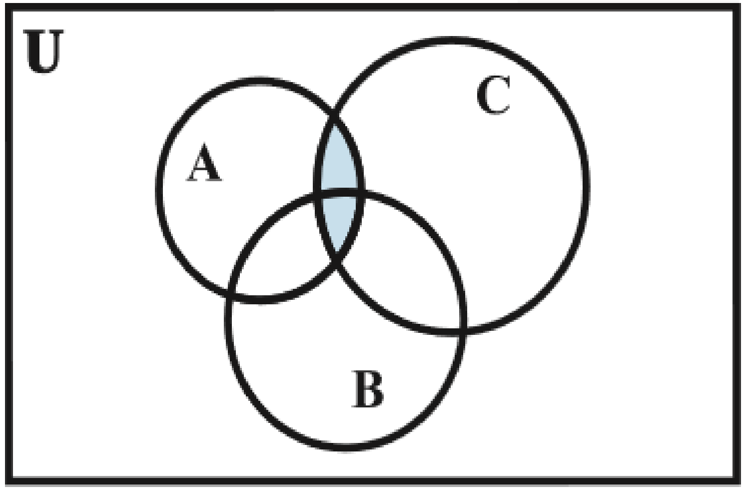
$B \cup C$



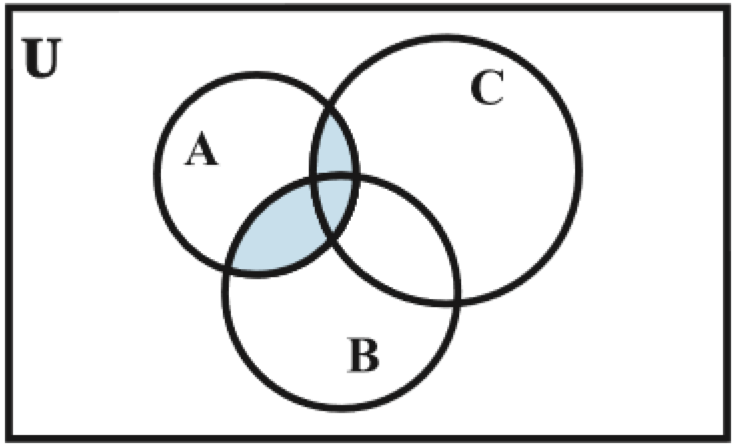
$A \cap (B \cup C)$



$A \cap B$



$A \cap C$



$(A \cap B) \cup (A \cap C)$

Practical problems on Union & Intersection of sets

- **if A and B are finite sets, then**

  $$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

- **If $A \cap B = \varphi$, then,** $n(A \cup B) = n(A) + n(B)$

- **If A, B and C are finite sets, then**

  $$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

Example

In a school there are 20 teachers who teach mathematics or physics. Of these, 12 teach mathematics and 4 teach both physics and mathematics. How many teach physics ?

Solution:Let M denote the set of teachers who teach mathematics and P denote the set of teachers who teach physics. Then,

$n(M \cup P) = 20$, $n(M) = 12$, $n(M \cap P) = 4$, $n(P) = ?$

$n(M \cup P) = n(M) + n(P) - n(M \cap P)$,

we obtain

$20 = 12 + n(P) - 4$ Thus $n(P) = 12$

Hence 12 teachers teach physics.

What have we learned today?

- Intervals are subsets of R.
- The power set P(A) of a set A is the collection of all subsets of A.
- The union of two sets A and B is the set of all those elements which are either in A or in B.
- The intersection of two sets A and B is the set of all elements which are common in A and B.
- If A and B are finite sets then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- If $A \cap B = \varphi$, then $n(A \cup B) = n(A) + n(B)$.

THANK YOU!